AIAA 80-0703R

Gust Response of Rotor and Propeller Systems

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The influence of nonstationary turbulence on rotor and propeller systems is discussed. The review is made from a common analytical basis of nonstationary approach, with emphasis on concepts rather than on details. The necessity of such an approach and its feasibility for predicting a complete set of gust and response statistics together with correlations with somewhat limited test data are appraised.

Introduction

DURING the past 10 years, considerable experience has been accumulated in treating nonstationary gusts and vibrations of flight vehicles. Three types of vehicles where such gusts are of particular significance should be mentioned. The first type pertains to aircraft flying through turbulence of varying intensity and experiencing overloads of transient excursions and high thresholds.[1] The second and the third type pertain, respectively, to lifting rotors[2-9] (including rotary fixed-wing aircraft and stowed rotors) and to large-diameter propellers in V/STOL and tilt-rotor airplanes.[10-14] In transition and forward flight conditions parameters of lifting rotors and propeller systems are time variant.[2-14] A given stationary fluctuation in gust velocity in the rotor or propeller disk produces quite different loads, depending on the azimuth position of the rotor or propeller blades.[2-14] Although the configurations of these vehicles differ rather significantly, the nonstationary analysis applies to all of them when accounting for gusts inducing nonstationary vibrations.

The predecessor to this nonstationary analysis is the stationary analysis or power spectral density (PSD) technique whose application is now a routine step in the design of aircraft to account for gusts and of VTOL vehicles for stationary random disturbances during deceleration after touchdown.[1,4,15] For airplane loads and structural dynamics analysis, this step was taken during the mid-1950s with the introduction of concepts from the stationary random process theory and of turbulence models available from the basic works of Taylor and von Kármán.[1,2,15]

A crucial question is whether the nonstationary approach will find acceptance in a routine dynamic analysis, as has been the case with the PSD technique. It is instructive to approach this question in three phases: 1) the necessity for complementing and extending the stationary approach; 2) its computational feasibility in obtaining a complete set of gust and response statistics—mean square values, threshold crossing, and peak statistics; and 3) the benefit of test results correlating with predicted gust and response statistics. The present state-of-the-art and further research concerning these three phases form the subject of this survey. As a prelude, general considerations are touched upon here, the earlier discussion concerning the first phase or necessity.

Computability, the second phase, is feasible for three reasons:

1) The nonstationary gusts can be described, *as in the stationary case,* on the basis of the Taylor-von Kármán hypothesis according to which the gust field is momentarily frozen with respect to time variations.[1,2,11,16] In other words, the vehicle passes through the turbulent eddies so fast that the effects of time changes of the eddy velocities on the vehicle can be neglected as compared to the effects of the changes in turbulence velocities along the flight path. This concept of a time-wise locally frozen gust field views the flight vehicle as passing through the random standing waves of turbulence velocities much as it rolls over a randomly bumpy runway. Most fundamental in the development of nonstationary analysis is that it permits dual conversion—time histories of turbulence measured from vehicles can be converted into spatial records and vice versa.[1,16]

2) The gust excitations, although nonstationary, are approximately Gaussian and the vehicles, although time variable for certain flight regimes, are linear. Therefore the responses are also Gaussian.[17] *As in the stationary case,* Rice's equations become applicable to evaluate threshold crossing and peak statistics.[17]

3) Gust excitations for all the three types of vehicles can be reasonably idealized as a separable nonstationary process in that the conventional stationary excitation is modulated by a deterministic function.[1,11,18] The designing of shaping filters to stationary processes is established and routine.[19] *As in the stationary case,* closed-form expressions of threshold crossing and peak statistics are available in the literature.[1,18-22] Therefore the treatment of nonstationary gusts and responses is no more involved than treating responses by the PSD technique in combination with sharp edge gusts, ramp gusts, one-minus-cosine gusts, etc.

The final or the third phase concerns correlation with test results. The PSD technique has the support of extensive full-scale and model tests. And a routine test process has emerged.[1,2,15] It is an outcome of extensive research of well over 25 years.[1,2,15] By comparison, measurements of nonstationary gusts and responses are recent and not extensive and have been made mostly through model tests. However, several innovative test procedures have been developed during the past 10 yr from which a routine test process is likely to evolve. This is true, for example, for the periodic mean square response[3] and flapping autocorrelation of articulated rotor blades[23,24] for the time-averaged power spectra of V/STOL propeller side forces and pitching moments,[25] for the spectral description of blade and wing responses of hingeless and gimballed rotor-propeller configurations,[8,10,27] for the alleviation of response excursions through active and passive controls in hingeless rotors and tilt-rotor aircraft,[8,10,26-28] for the rms description of response derivatives with the inclusion of pilot contribution in rough-weather-transport helicopters,[9] etc. These tests offer promise of providing answers to questions not presently within the validity regime of the extensive analytical background developed to date.

Modeling Nonstationary Gusts

With reasonable approximation, nonstationary gusts can be modeled as belonging to a separable nonstationary process in

Presented as Paper 80-0703 at the AIAA/ASME/ASCE/AHS 21st Structures, Structural Dynamics & Materials Conference, Seattle, Wash., May 12-14, 1980; submitted June 12, 1980; revision received Feb. 18, 1981. Released to AIAA to publish in all forms.

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that a stationary random process is modulated by a deterministic function.[1,29-31] As an illustrative example with reference to nonstationary vertical turbulence velocity $W(t)$, the modeling equation is

$$W(t) = g(t)G(t) \tag{1}$$

where $g(t)$ is the input modulating function and $G(t)$, the conventional vertical turbulence velocity which is stationary and Gaussian. By stipulation, $G(t)$ has a zero mean value. It is described by its autocorrelation $R_G(t)$ or equivalently by its spectral density $S_G(\omega)$ which is the Fourier transform of $R_G(t)$. The model governed by Eq. (1) is also referred to as the intensity modulated or uniformly modulated[1] nonstationary process which finds extensive applications in several fields. The related basic literature on this model is rather extensive. Of particular relevance to gust-response studies are Refs. 31-37 which provide its stochastic structure in time, frequency, and mixed domains. References 38 and 39 concern algorithmic details of evaluating Fourier coefficients when $g(t)$ is expressed as a Fourier series. References 32, 33, and 40 touch upon experimental aspects of measuring time-dependent spectra, and Refs. 41-45 pertain to its influence on response structure. Since the modulation according to Eq. (1) is a subset of linear operations on $G(t)$, the nonstationary gust $W(t)$ is also Gaussian. The autocorrelation or ensemble average of $[W(t_1)W(t_2)]$ is

$$E[W(t_1)W(t_2)] = R_{WW}(t_1,t_2) = g(t_1)g(t_2)R_{GG}(t_2-t_1) \tag{2}$$

from which the variable mean square value with $t_1 = t_2 = t$ reduces to

$$\sigma_W^2(t) = [g(t)]^2\sigma_G^2 \tag{3}$$

where σ_G is the rms value of $G(t)$.

By definition, the turbulence scale length L is given by

$$L = 2/\sigma_G^2 \int_0^\infty R_G(x)\,dx \tag{4}$$

which is twice the length of $G(t)$. In the PSD technique the stationary turbulence velocity $G(t)$ is described by its spectral density function $S_G(\omega_r)$ of which three models are in use. Usually the preferred one is the von Kármán model[6,15]

$$S_G(\omega_r) = \sigma_G^2 \frac{L}{2\pi} \frac{1 + \frac{8}{3}(1.34L\omega_r)^2}{[1 + (1.34L\omega_r)^2]^{11/6}} \tag{5}$$

where ω_r is the spacewise circular frequency and $\omega_r = 2\pi k$, k being the wavenumber per unit length. Considerable simplification of the analysis without appreciable sacrifice in accuracy is feasible by having exponential curve representations for the correlation function.[1,4] Therefore in several studies the spectral density [Eq. (5)] is approximated by rational spectra as a ratio of two polynomials in ω_r^2. Two such expressions for $S_G(\omega_r)$ are

$$S_G(\omega_r) = \sigma_G^2 \frac{L}{2\pi} \frac{[1 + 3(L\omega_r)^2]}{[1 + (L\omega_r)^2]^2} \tag{6}$$

and

$$S_G(\omega_r) = \sigma_G^2 \frac{2L}{\pi} \frac{1}{[4 + (L\omega_r)^2]} \tag{7}$$

where Eq. (6) is the well-known Dryden spectrum used in most of the nonstationary gust-response studies of airplanes.[1,15] Equation (7) is the Ornstein-Unlenbeck spectrum used in most of the gust-response studies of rotorcraft.[4,16,18-22] In Fig. 1, Eqs. (5-7) are depicted for $L = 152$ and 600 m, which will be referred to in subsequent presentations of numerical data.

Gust Excitation

Hingeless rotorcraft without feedback systems or without large horizontal tails are gust sensitive: the higher the advance ratio, the greater the sensitivity.[6] It is generally recognized that the present gust-load criteria such as MIL-S-8698(ARG) are conservative[46,47] and that dynamic loads from gusts are of importance when meeting ride comfort standards of commercial helicopters.[46] Reference 9 also considers the commercial use of helicopters for transport services of restricted access such as the inter-rig services on off-shore oil platforms in the North Sea and the lighthouse-type services operating to and from landing pads, some built on top of the lantern. It shows that such extreme weather operations warrant a detailed prediction of gust performance and that turbulence is a "leading factor" when assessing pilot workload, passenger comfort, and structural loadings. Generally for conventional low-speed articulated rotors which are highly loaded, dynamic loads from level flight cruising and maneuvering (including unsteady aerodynamics) are usually adequate for dynamic loads from gusts.[2] However gust loads become predominant for unloaded rotor configurations as in compound helicopters with rotor off-loading by a fixed wing, stowed rotors during

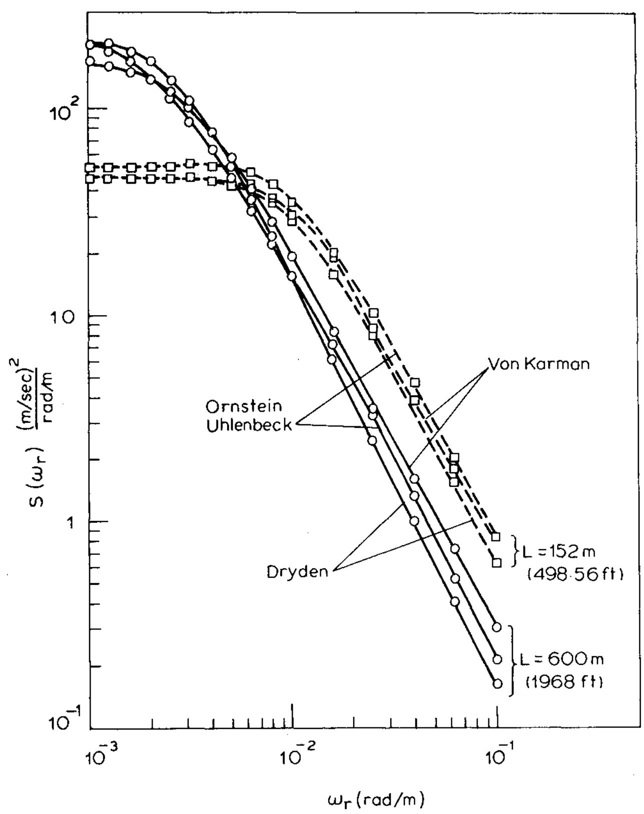

Fig. 1 Comparisons of power spectral densities of stationary gusts used in nonstationary analyses.

folding and stowing operations, and rotors in tilt-rotor airplanes.[2,21] Except in a few restricted cases of hovering,[23,48,49] gusts on the rotor disk cannot be directly treated by the PSD technique. Even those treatments of a few cases are based on the stipulation that a viable turbulence theory is available for hovering and near-hovering modes. The Taylor-von Kármán turbulence model is based on the assumption of a large constant relative mean flow velocity as compared to the turbulence velocities. A rms severity of 0.305 m/s vertical turbulence velocity occurs at altitudes between 122 and 213 m above terrain with about 0.1% probability.[50] The concept of frozen random space waves used in this model does not allow a plausible extension to the hovering rotor.[50] In order to be applicable, the relative wind velocity must be large as compared to the rms severity of turbulence velocities. As a matter of fact, such an application is likely to give infinitely slow timewise variations of turbulence velocities at a point in the atmosphere.[50] A similar situation occurs in the gust response of V/STOL airplanes near the hovering mode.[1] However, the observation that the Taylor-von Kármán turbulence theory is inapplicable to a hovering helicopter is not without controversy. As such the development of a viable turbulence model for V/STOL vehicles near hovering warrants immediate attention.

By comparison with airplanes, turbulence flow in the rotor disk is relatively more involved. For example, when nonuniformity is included, gust excitations as experienced by different blade stations belong to a complete nonstationary process in addition to being modulated by deterministic periodic functions.[16,50-54] An insight into the stochastic structure of turbulence in the rotor plane and also to some extent in the propeller plane can be gained by studying the autocorrelation function $R_w(t_1,t_2)$ at, say, a representative blade station $0.7R$ from the center (see Fig. 2). Such a study is also important in assessing the viability of approximations to completely nonstationary models. Note that the development of approximate models has a fundamental bearing on the very acceptability of nonstationary treatment of rotor and propeller responses to gusts.

According to Taylor's hypothesis, the autocorrelation function depends only on the spatial separation r.[1,16-18] Therefore

$$R_w(t_1,t_2)=g(r) \tag{8}$$

where

$$r=[\{X(t_2)-X(t_1)\}^2+\{Y(t_2)-Y(t_1)\}^2]^{1/2} \tag{9}$$

With respect to the spectral density given by Eq. (7), the autocorrelation is given by

$$R_w(t_1,t_2)=\sigma_w^2\exp(-2|r|/L) \tag{10}$$

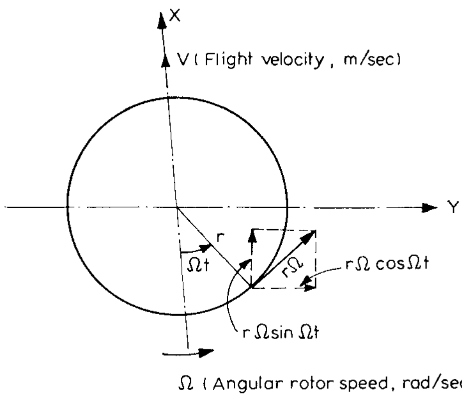

Fig. 2 Components of blade velocity at a blade station with radius $r=0.7R$.

As seen from Fig. 2, the velocities as experienced by the $0.7R$ station are

$$\frac{dx}{dt}=V+0.7R\Omega\sin\Omega t \tag{11a}$$

$$\frac{dy}{dt}=0.7R\Omega\cos\Omega t \tag{11b}$$

The spatial separation r evaluated from Eqs. (11) is a function of both t_1 and t_2. With reference to the exponential autocorrelation function as in Eq. (10), and with the definitions of the advance ratio $\mu=V/\Omega R$ and of the rotor turbulence parameter $a=2\mu/(L/R)$, the autocorrelation function for the dimensionless inflow or $W/\Omega R$ reduces to[18]

$$R_\lambda(\bar{t}_1,\bar{t}_2)=\sigma_\lambda^2\exp(-|[\{a(\bar{t}_2-\bar{t}_1)-1.4(R/L)\cos\bar{t}_2+1.4(R/L)\cos\bar{t}_1\}^2+\{1.4(R/L)\sin\bar{t}_2-1.4(R/L)\sin\bar{t}_1\}^2]|^{1/2}) \tag{12}$$

where $\bar{t}$ is the dimensionless time Ωt. When nonuniformity is neglected in both the longitudinal and lateral directions, the velocity is V in the flight direction and the correlation distance simplifies to the stationary case

$$r=V(t_2-t_1)=\mu R(\bar{t}_2-\bar{t}_1) \tag{13}$$

Equation (13) is extensively used in lifting rotor and propeller gust-response studies.[2,16] Physically, it says that the gust velocities as they exist at the rotor center are taken as representative of the entire rotor disk, or what is referred to as the point approximation in propeller gust-response studies.[11,16,25] The corresponding stationary autocorrelation function with respect to Eqs. (9) and (11) simplifies to

$$R_\lambda(\bar{t}_1,\bar{t}_2)=\sigma_\lambda^2\exp(-|a(t_2-t_1)|) \tag{14}$$

whose spectral density expression as given by Eq. (7) is depicted in Fig. 1. In Refs. 50 and 51 nonuniformity is considered only in the flight direction such that $dy/dt=0$ in Eq. (11b), and the corresponding autocorrelation function takes the form

$$R_\lambda(\bar{t}_1,\bar{t}_2)=\sigma_\lambda^2\exp\{-|a(\bar{t}_2-\bar{t}_1)-1.4(R/L)\cos\bar{t}_2+1.4(R/L)\cos\bar{t}_1|\} \tag{15}$$

The turbulence as in Eq. (15) can be treated as modulated nonstationary in that it can be simulated by passing the stationary turbulence typified by Eq. (14) through a linear delay-type filter.[18,50-52] Equations (12), (14), and (15) demonstrate that within the framework of correlation theory, the structure of turbulence in the rotor plane for a given advance ratio μ is determined by L/R, the ratio of turbulence scale length over rotor radius R. It should also be noted that incorporating Taylor's hypothesis as in Eq. (8) is equivalent to assuming that the rotor disk turbulence is essentially represented by free atmospheric turbulence. Such an assumption is reasonable for "low-lift/high-advance ratio" rotor operations when the effects of self-induced turbulence are likely not to be substantial.[18,20,50-52] Such operations are typical of high-speed rotorcraft with rotor off-loading by a fixed wing. It should also be observed that Taylor's hypothesis also permits converting measured time histories into space histories and vice versa. Recent experimental studies on propellers operating in turbulence support this hypothesis.[25] However, mention must also be made of two exploratory studies[53,54] in which the autocorrelation function is expressed in terms of both time and space coordinates.

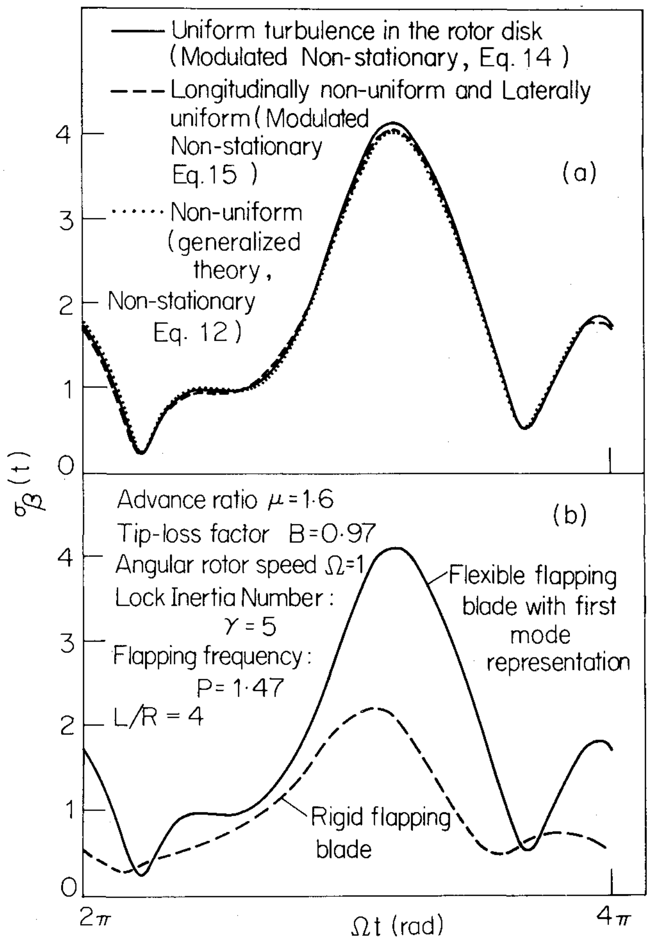

Fig. 3 Effects of gust nonuniformity and mode shape on flapping response.

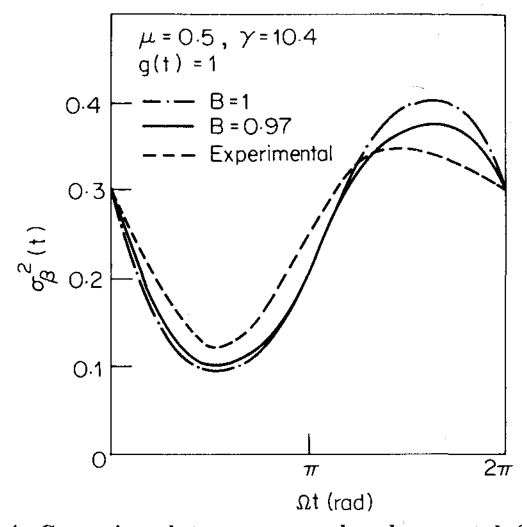

Fig. 4 Comparison between measured and computed flapping response variance.

Computing Response Statistics

In addition to rms descriptions of response components, vibration excursion and fatigue studies require threshold crossing and peak statistics.[1,17] Once the response variance matrix is known, a complete set of response statistics can be evaluated from the closed-form formulas.[1,18-22] Rotor propeller systems in forward flight have periodic coefficients and the input processes belong to a nonstationary process.[56] As such, computing the corresponding response variance matrices is not as routine as in the PSD technique. For completely nonuniform or nonstationary turbulence as in Eq. (12), the design of shaping filters is involved.[19] Therefore the response variance matrices are generally computed by introducing the state transition matrices of the adjoint systems.[18,50] For nonstationary turbulence as in Eq. (15), which represents stationary turbulence modulated by a delay-type linear operator L^*, the frequency domain approach is used in which the stationary input is represented by the spectral density description and the system by the "frequency response" matrix to the delayed input $L^*\exp(i\omega t)$.[18,50,51,55,57] For the cases of stationary and modulated nonstationary gust excitations—stationary gust modulated by deterministic functions—the shaping-filter approach is used in which the stationary input is passed through a filter and the response variance matrix is obtained directly as the solution of a matrix differential equation.[19,50-52,58-61]

In Fig. 3a (from Ref. 18) the flapping response rms values are shown for three types of turbulence models typified by Eqs. (12), (14), and (15). The advance ratio $\mu = 1.6$, $L/R = 4$, and $\sigma_\lambda = 1$. The blade model refers to a flexible flapping blade with a first-mode representation.[62] As seen from Fig. 3a the nonuniformity of turbulence in the rotor plane has negligible effect. Even for the largest lifting rotors of the next generation of rotorcraft, and for small turbulence scale lengths encountered at low altitudes, the stationary model is shown to be adequate. The restriction noted earlier that the rotor self-induced turbulence must be negligible is still required and limits the probabilistic model to "high-advance-rato low-thrust" conditions. This finding that the effects of nonuniform turbulence over the rotor disk are negligible is of significance for two reasons: 1) gust excitations for rotors can be modeled as separable nonstationary according to Eq. (1), as used in airplanes; and 2) the input modulating functions and the system parameters being periodic, under steady state both the input and output processes can be characterized as periodic nonstationary processes. Consequently the gust-response analysis becomes "quite similar to the well known analysis of time invariant systems subject to stationary excitations."[56] Henceforth, all the numerical data will be presented with the stipulation of point approximation or uniform turbulence in the rotor or propeller plane. For $\mu = 1.6$ and $L/R = 4$, Fig. 3b (from Ref. 62) shows that with the inclusion of blade flapping flexibility, the global maximum of flapping rms value almost doubled. Similarly, in Ref. 63 the effects of including torsional flexibility to a rigid flapping blade are studied. This study shows that blade torsional response to atmospheric turbulence at high advance ratios can be very severe unless the torsional blade stiffness is several times greater than that for static torsional divergence limit in the region of maximum reversed flow.

In Fig. 4 (from Refs. 3 and 64), the flapping variance of a centrally hinged rigid blade with flapping frequency $P \simeq 1$ is compared with the experimentally measured data of Ref. 3 for $\mu = 0.5$ and Lock number $\gamma = 10.4$. The input process is close to being a stationary white noise without modulation that was generated in the wind tunnel by placing a grid of bars perpendicular to a uniform airflow. Depending upon the diameter of the grid bars and their spacing, an approximately homogeneous and isotropic turbulence is generated.[3] In a sufficiently downstream position turbulence acts as a low-pass type of white process with an almost flat spectral density function. The measured data correlate well with the computed values. The consideration of 3% tip loss with $B = 0.97$ appears to give better correlation. This noteworthy experimental setup[3] is probably the first to measure nonstationary random responses of rotor blades.

From the viewpoint of ride qualities and fatigue, threshold crossing and peak distribution data are required.[17,18] Basically, the problem boils down to assessing how far the response process is narrowband.[65,66] Since both the input and output processes are periodically nonstationary, it is possible to define the peak distribution over one period or rotor revolution that is independent of the instantaneous blade azimuth locations. As in Refs. 18,22,65, and 66, the distribution function of β over one rotor revolution can be defined as follows:

$$F_{\beta T}(\xi) = \text{probability } \{\text{any maxima of } \beta \leq \xi \text{ in one rotor revolution}\} = 1 - \left(\int_0^{2\pi} E[M_\beta(\xi,t)]\,dt \Big/ \int_0^{2\pi} E[M_{\beta T}(t)]\,dt \right) \tag{16}$$

In the above equation, $E[M_\beta(\xi,t)]$ is the average number of flapping peaks above ξ per unit time and $E[M_{\beta T}(t)]$, which is equal to $E[M_\beta(-\infty,t)]$, represents the average total peak rate irrespective of magnitude. According to the narrowband approximations[18,22,65,66]

$$\int_0^{2\pi} E[M_\beta(\xi,t)]\,dt \simeq \int_0^{2\pi} E[N_{+\beta}(\xi,t)]\,dt$$

and

$$\int_0^{2\pi} E[M_{\beta T}(t)]\,dt \simeq \int_0^{2\pi} E[N_{+\beta}(0,t)]\,dt \tag{17}$$

where $E[N_{+\beta}(\xi,t)]$ represents average number of flapping upcrossings of threshold ξ per unit time. The data in Figs. 5 and 6, both from Ref. 18, depict different facets of this narrowband structure of response for the flap-bending blade presented in Fig. 3. Figure 5 shows that the flapping response is approximately narrowband in that the average number of peaks per unit time above the zero mean level is close to the total number of peaks per unit time regardless of magnitude. The data in Fig. 6 show that the approximations according to Eq. (17) are indeed satisfactory except for very low thresholds which are not of much practical significance.

For gaining more understanding of the periodic response process, the data in Fig. 7 (from Ref. 50) are presented from an analog study for $\mu = 1.6$, $\gamma = 4$, $P = 1.3$, and $L/R = 4$. Given the crudeness of a simulation covering only 1000 periods with a discretization of 15 deg azimuth intervals, the computed rms flapping values agree fairly well with analog results. Of interest is the sample function of β in Fig. 7 from which it is seen that the response process is close to being a narrowband. This finding is of practical significance in two respects: First, the computations of response peak statistics basically reduce to evaluating threshold crossing expectations for which computationally simpler expressions are available (for details, see Refs. 1, 18-22); second, further development of stochastic treatment of ride qualities and fatigue requirements becomes relatively easy to perform.

Mention must also be made of two pioneering studies[7,67-69] in which the effects of atmospheric turbulence on the stability of rotor blades are studied. The physics of the model is identical to the one treated in Ref. 63, but the equations of motions (rigid flapping and elastic torsion) are derived in a stochastic turbulent environment which leads to stochastic terms in the homogeneous equations as parametric excitations.

The preceding nonstationary treatment of rotors lends itself well to the prediction of random loads and vibrations of large-diameter propellers in axial flow subject to atmospheric turbulence.[11,16,25] References 11 and 16 discuss the problem of forces and moments on a propeller in axial flow due to gusts based on the point approximations referred to earlier. Figure 8 (from Ref. 11) gives the time-averaged power spectral density of the side force of a single-bladed propeller. It has spikes at the rotor frequency of revolution and at twice this value. These spikes disappear for a three-bladed (or more) propeller, since the equations of blade motions written in multiblade coordinates have almost constant coefficients.

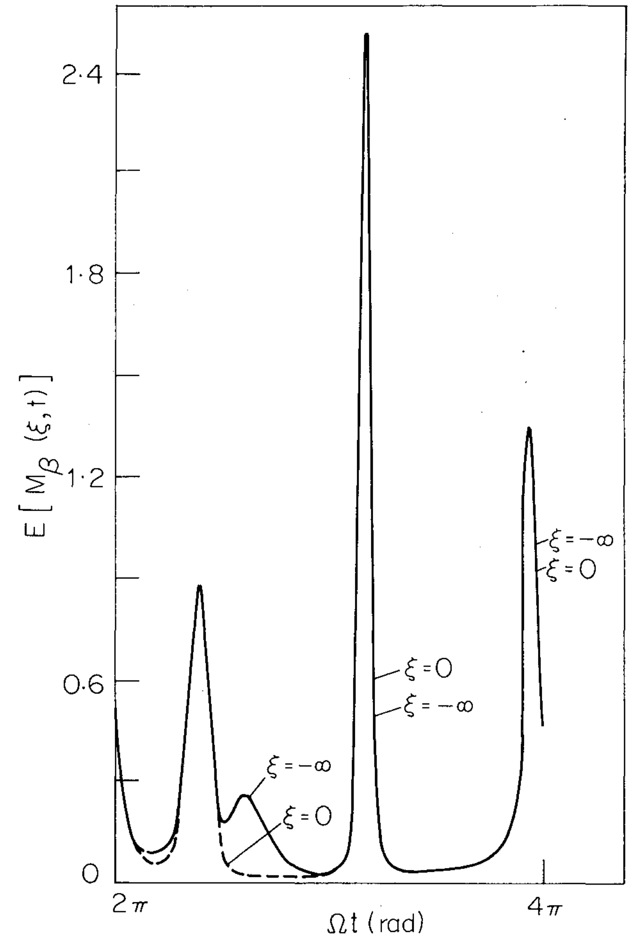

Fig. 5 Average number of flap-bending peaks per unit time irrespective of magnitude and above zero mean level (data as in Fig. 3).

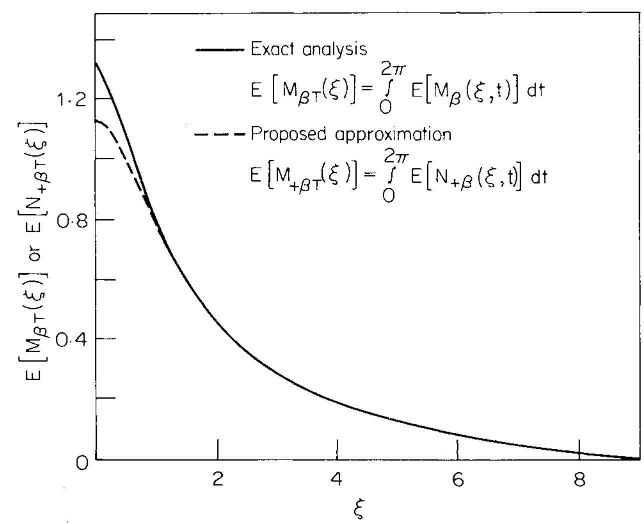

Fig. 6 Narrowband features of response process over one rotor revolution.

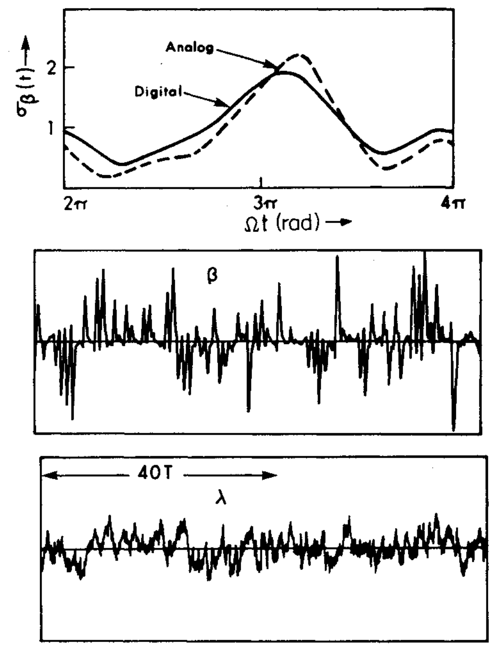

Fig. 7 Comparison with simulation studies and sample functions of stationary inflow and nonstationary response.

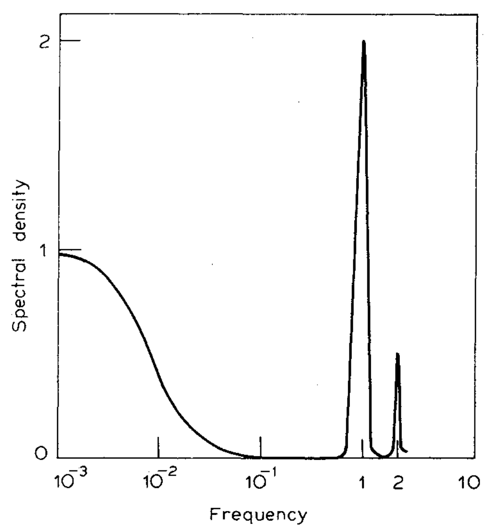

Fig. 8 Time-averaged power spectral density for side force of a single-bladed propeller.

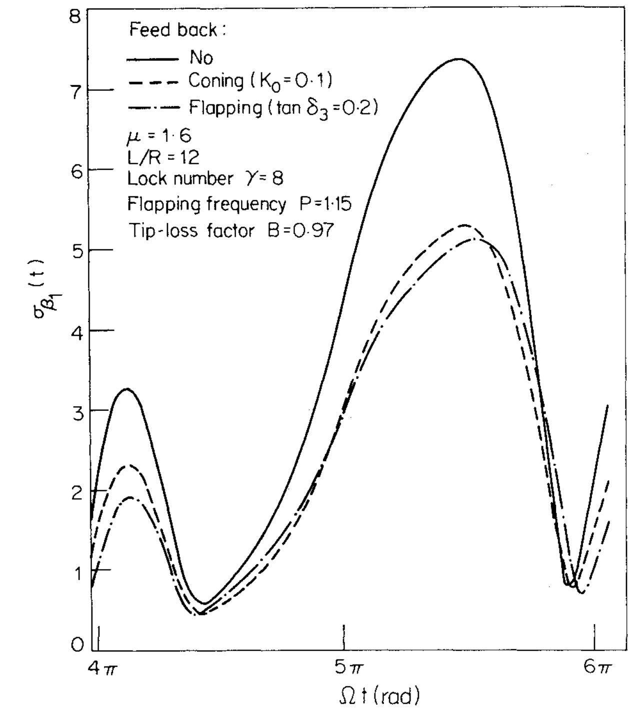

Fig. 9 Effects of coning and flapping feedback in alleviating gust sensitivity.

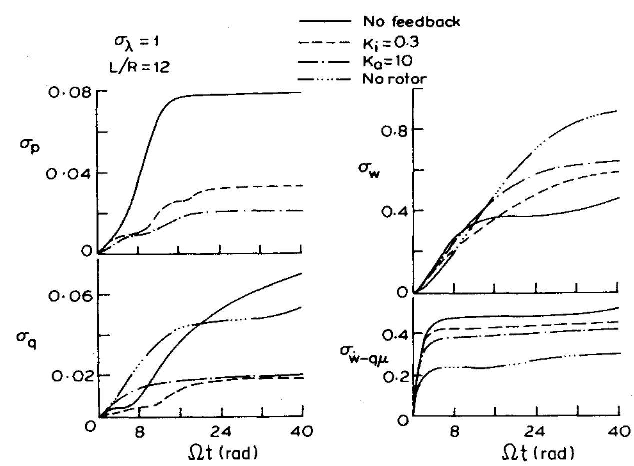

Fig. 10 Effects of normal acceleration and rotor tilting feedback systems on gust responses of a hypothetical compound rotorcraft, $\mu = 0.8, \gamma = 5$.

Experimental results of Ref. 25 give reasonable correlation with predicted spectral results. This is noteworthy, given the difficulty of simulating artificial wind-tunnel turbulence with scale length much larger than the propeller disk radius as warranted by the point approximation.[10,26,28] Tilt-rotor airplanes with either hingeless or gimballed rotors have greater gust sensitivity than conventional small-diameter propeller aircraft.[26] They are generally analyzed with the use of multiblade or nonrotating coordinates[70,71] when the system response and gust inputs can be treated conceptually within the framework of stationary analyses.[10,28,72-75] It should be emphasized that both the input and response processes are nonstationary in the rotating frame.

Gust-Response Control

For both military and commercial rotor and propeller systems it becomes necessary to alleviate gust-response excursions to improve fatigue life, pilot and passenger comfort, target delivery, etc. Accordingly, controlling gust responses has been extensively researched. A description of feedback systems for reducing gust sensitivity is given in Refs. 6,19,52, and 63 for helicopters with hingeless rotors and in Refs. 12, 26-28, and 72-74 for tilt-rotor airplanes with hingeless or gimballed rotors.

The data in Fig. 9 (from Ref. 19) describe a three-bladed rotor system at an advance ratio of 1.6. The system parameters are: Lock number $\gamma = 8$, the rigid flapping frequency $P = 1.15$, and tip loss factor $B = 0.97$. The gust input is characterized by $L/R = 12$ and $\sigma_\lambda = 1$. Coning feedback with gain 0.1 and flapping feedback (pitch-flap coupling) with gain 0.2 are considered. It is seen that absolute maximum of the flapping rms value of the first blade or $(\sigma_{\beta_I})_{max}$ is reduced by about 30% with inclusion of either of these two feedback controls. These two feedback systems when applied to a coupled torsion-flapping rotor system are found to be effective in alleviating only flapping, with little effect on torsion.[63]

Figure 10 (from Ref. 75) concerns rigid-body roll p, pitch q, vertical motion w, and the normal acceleration $\dot{w} - q\mu$ for a hypothetical hingeless compound helicopter at an advance ratio of 0.8. The transient conditions are initiated with $t = 0$ when the helicopter just enters turbulent patches with $L/R = 12$ and $\sigma_\lambda = 1$. The control gains k_i and k_a pertain, respectively, to rotor tilting and normal acceleration feedback systems. While the rotor has a destabilizing effect, the feedback systems have an appreciable stabilizing effect. It is significant that both the control systems effect a large reduction in random pitch rate response when compared even to the stable configuration without the rotor. Reference 8 includes theoretical and experimental investigation of alleviating gust sensitivity by shifting the chordwise center of gravity. Theoretical results agree well with data from an experiment which is innovative with respect to achieving an appropriate shift of the chordwise center of gravity either fore or aft and with respect to simulating flapping, lag, and torsional frequencies. Although in a strict sense there is a flapping hinge, the flexural restraint is so adjusted that the flapping frequency is typical of hingeless rotors. While such an approach of shifting the chordwise center of gravity is an effective method of alleviating low-frequency flapping modes to vertical gusts, it has a deteriorating effect on high-frequency torsional modes.[8]

We also mention a "fly-by-wire" feedback system[76] to improve rms response of rotors subject to turbulence, which could also find similar applications to tilt-rotor airplanes. According to this feasibility study,[76] an onboard von Kármán type of filter/controller can be used with the novelty of feeding back all of the state variables into the controls while only a few state variables are actually being measured.

Conclusion

There exists voluminous literature on the stochastic models of nonstationary atmospheric turbulence and corresponding response treatments of rotor and propeller systems. Generally, each of these models describes a single isolated problem and is based on a variety of simplifying assumptions and approximations. An appraisal of these models and response treatments shows that the state-of-the-art is established with respect to computing a complete set of gust and response statistics of design interest such as rms values, threshold crossing, and peak distributions. It demonstrates the necessity of treating nonstationary gusts and responses by the nonstationary approach. However, with respect to the feasibility of these models in a routine dynamic analysis, it leads to the following remarks:

1) There are large gaps in our knowledge of gust encounters during hovering or near-hovering modes when the vehicle changes speed over distances that are not large compared to turbulence scale lengths. There still is much research needed for a viable turbulence theory for the hovering helicopter and for all V/STOL vehicles in the near-hovering mode.

2) Turbulence is of primary importance for future high-speed lifting-rotor VTOL system used for commercial purposes. The finding that the nonuniformity effects of turbulence in the rotor plane are negligible is of practical significance. However the basic assumption leading to this finding is that the self-induced turbulence is negligible compared to the free atmospheric turbulence. Such an assumption needs to be checked out in full-scale and model tests. This will require some innovations in test technology since no testing has yet been performed with artificially produced nonstationary turbulence with scale length much larger than rotor radius.

3) The finding that the rotor response process belongs to a narrowband process is also of practical significance. It offers promise in that the further development of stochastic treatment of ride qualities and fatigue requirements becomes relatively easy to perform.

Acknowledgments

The author would like to thank K. H. Hohenemser, Washington University, St. Louis; Y.K. Lin, University of Illinois, Urbana; and N. D. Ham, Massachusetts Institute of Technology, Cambridge, for discussions. Thanks are also extended to C.V. Joga Rao, Indian Institute of Science, Bangalore, for acquainting the author with his valedictory address, "Gust Response of Helicopters," delivered on Dec. 17, 1977, in connection with a short course on "Random Vibration—Theory and Applications."

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